Abstract—WiMAX is one of the most promising technologies to provide broadband wireless access in the near future. In this paper we identify a key element for the performance of a WiMAX network, the DL-MAP packing algorithm, which mainly determines the usage efficiency of the available radio resources and investigate potential differences that could appear between WiMAX equipment vendors in the maximum capacity of the system due to the packing approach used. Our results show that the performance of simple DL-MAP packing algorithms might be significantly outperformed by more complex ones resulting in a clear differentiation factor among manufacturers.

Index Terms—WiMAX, OFDMA, QoS and DL-MAP.

I. INTRODUCTION

The popularity of the Internet is constantly pushing the evolution of access technologies in order to provide larger bandwidth as well as seamless mobility support at a lower cost. In this context, the IEEE 802.16 family of standards, 802.16-2004 [1] and 802.16e-2005 [2], has been developed to address these needs. The 802.16 technology aims at providing broadband wireless access and promises throughput rates and coverage distances which should clearly outperform the ones of currently well-established technologies such as 3G and Wireless LAN. Interoperability among vendors is guaranteed by the WiMAX Forum [3] which defines the certification programs for this technology.

The main enhancements that WiMAX offers as compared to its competitors comes from the advanced physical layer technology incorporated which includes the latest developments in the field, e.g., S-OFDMA and MIMO. The large degree of flexibility offered by this promising technology though, requires of careful engineering in order to exploit all its potential. One of the key differentiators of WiMAX systems is the combined performance of the Base Station QoS scheduler and the DL-MAP packing algorithm which determine whether the agreed QoS guarantees are honoured and how efficiently the available radio resources are used. Our focus in this paper is the study of the impact of a WiMAX base station DL-MAP packing algorithm on the overall system performance in order to provide guidance on the trade-off to be considered between complexity and associated performance improvement.

The rest of the paper is structured as follows. In the next section we describe the tasks to be performed by a WiMAX base station QoS scheduler and a DL-MAP packing algorithm pointing out the major challenges to be addressed. In Section III a brief review of related work for DL-MAP packing algorithms is provided and three examples described plus an ideal one that will be used as performance upper bound reference. A quantitative analysis of the potential performance differences between simple and more complex DL-MAP packing algorithms is provided in Section IV. Finally, Section V summarizes the results and concludes the paper.

II. RADIO RESOURCES USAGE IN WiMAX

A WiMAX network is composed of Subscriber Stations (SS) and Base Stations (BS). Base stations fully control the access to the air interface, both in the uplink and the downlink direction, and carry out diverse functions such as handling QoS provisioning, traffic classification, tunneling of information and encryption in the WiMAX cell. Subscriber stations adhere to the instructions mandated by base stations. An overview of the IEEE 802.16-2004 standard can be found in [4].

In order to exchange information over the air, the 802.16 family of standards offers different pos-
sibilities being time division duplexing (TDD) in combination with OFDMA the choice currently favored by most implementations. The exchange is based on WiMAX frames which are arrangements of OFDMA symbols in time and subcarriers in frequency. A frame for a WiMAX TDD OFDMA system is illustrated in Figure 1. In TDD, a frame is divided into downlink and uplink subframe. The downlink subframe consists of a preamble followed by a frame control header and downlink (DL-MAP) and uplink maps (UL-MAP). The maps are used to indicate to the SSs where to find information addressed to them in the downlink or when to transmit information in the uplink.

In the downlink, base stations place incoming MAC protocol data units in rectangular areas called bursts. The name burst comes from the fact that each rectangle can be transmitted using a particular modulation and coding scheme referred to as burst profile. The rectangular shape restriction does not exist for uplink bursts and this considerably simplifies their placement in the uplink subframe.

Frames can be constructed by permuting subcarriers in a variety of ways and then grouping them to create subchannels. For example, bursts inside a frame can be transmitted using non-adjacent subcarrier frequencies. This provides the system with means to counter frequency selective fading and under this scheme the subscriber stations experience a similar quality over any logical subchannel. The subcarrier groups are formed based on a set of predefined schemes. In WiMAX, one of such non-adjacent groupings is the partial usage of subcarrier (PUSC) mode.

The smallest atomic tile that can be assigned in a WiMAX frame for overhead or data is a slot. The definition of slot varies according to the subcarrier grouping scheme. For PUSC, in the downlink, a slot is an area formed by one subchannel by two OFDMA symbols. In the uplink it corresponds to one subchannel by three OFDMA symbols. Throughout the article we focus on the PUSC case, a mandatory mode in WiMAX.

The selection of data to be placed in a downlink subframe is performed by a QoS scheduler. Such a scheduler should be aware of the QoS requirements of the flows in the system and schedule transmissions accordingly. The data selected is then placed into the downlink subframe according to the downlink MAP packing algorithm instructions. For each burst, the DL-MAP packing algorithm is in charge of computing its appropriate dimensions and location. The area of each burst is computed taking into consideration burst profile information provided by a channel monitor.

The overall performance of the system depends on the combined efficiency of the QoS scheduler and DL-MAP packing algorithm which try to maximize the radio resources usage while guaranteeing the required QoS to applications. In the following we describe some of the main challenges that need to be addressed in order to maximize the performance of the system:

**QoS provisioning:** In a WiMAX system QoS provisioning depends not only on the QoS scheduler but also on the DL-MAP packing algorithm. Because of the rectangular allocation of resources in the WiMAX downlink subframe, it might not be sufficient having enough resources in number of slots to transmit the data. It could happen that no solution is found that can pack all the data in rectangular bursts in the available capacity.

**Radio Resource Usage:** A DL-MAP packing algorithm that strictly tries to maximize the utilization of the WiMAX frame can result in a violation of the agreed QoS guarantees and/or unfair distribution of the resources. Therefore, a trade-off between radio resource usage maximization, QoS guarantees and fairness needs to be considered.

**Available Capacity:** The capacity available in the downlink subframe for data can not be known a priori since it depends on the outcome of the DL-MAP packing algorithm which has to fit the data in rectangular bursts. Moreover, the resources of the
downlink subframe used by DL-MAP also depends on the number of bursts packed at the end.

Packing Constraints: Because of design considerations some kind of rectangular shapes or positions in the WiMAX frame might be preferred, e.g., allocations in the subchannel dimension due to power consumption reasons. A DL-MAP packing algorithm should be able to adapt to packing constraints and still be reasonably efficient.

Computational Load: A common length of a WiMAX frame is 5ms considering both downlink and uplink subframes. Thus, in this case, 5ms is the maximum time that a WiMAX base station has, taking decisions on a per frame basis, in order to maximize performance. As a result, the computational load required by both the QoS scheduler and DL-MAP packing algorithm should be kept as low as possible to meet this tight deadline without requiring too expensive hardware.

Based on the aforementioned challenges, it can be observed that, while the task of a WiMAX base station QoS scheduler is similar to the one of QoS schedulers of other networking technologies, the task of a DL-MAP packing algorithm is very specific to the WiMAX OFDMA system. Therefore, in the rest of the paper we focus on the study of DL-MAP packing algorithms and their impact on the overall WiMAX system performance.

III. DL-MAP PACKING ALGORITHMS

The problem of packing rectangles inside a rectangular bin is a generalization of the one-dimensional bin-packing problem studied in [5]. Algorithms that place rectangular boxes inside rectangular bins have been thoroughly researched and are available in the literature, e.g., [6], [7]. However, these approaches assume that the rectangular dimensions of the boxes are fixed and then elaborate on the different packing methods to efficiently pack them in a bin. In WiMAX these conditions do not hold since the dimensions for each burst can be chosen by the packing algorithm as convenient. Additionally, padding can be added to the data to be packed in order to fit it inside a rectangular shape.

With regard to the OFDM technology, the issue of how to efficiently allocate resources has been studied for instance in [8] and [9]. In [8], the authors propose various mapping algorithms for OFDMA taking into consideration a priority mechanism useful in a QoS bounded environment. Fairness is explored in [9], where the authors derive boundary conditions and propose a heuristic approach to the allocation of frame resources. Both approaches though can not be applied to the WiMAX OFDMA downlink packing problem because they do not consider the rectangular allocation limitation.

In general, two main objectives should be considered for the design of a DL-MAP packing algorithm:

- Maximize the efficient utilization of the physical layer resources while meeting the required QoS guarantees and ensuring fairness.
- Minimize the complexity required for the packing algorithm while getting as close as possible to the optimum.

In the following we describe two examples of packing algorithms that could be applied to the WiMAX OFDM case plus an ideal one that provides an upper bound on the maximum performance that could be achieved.

A. Simple Packing Algorithm

As a first example of a possible WiMAX OFDM packing algorithm, we describe a simple packing algorithm that works in the following way. First, it receives a list of dequeued MAC protocol data units from the BS QoS scheduler and starts packing them in a First-In-First-Out (FIFO) manner. The packing is done such that it fills up the WiMAX downlink subframe by allocating whole columns from top to down (subchannel dimension) until the number of slots required for the amount of data is reached. If the number of slots required are not an integer multiple of the subchannel dimension the rest is filled with padding. Figure 2 illustrates an example of the packing layout produced by the simple packing algorithm.

The reason to choose an approach that always allocates whole columns is to simplify the packing problem. In this way it is very easy for the algorithm to deal with the leftover empty space since it just needs to keep track of the remaining number of symbols to determine if the next burst can be packed. Obviously, this packing algorithm is not very efficient since depending on the number of slots that need to be packed a lot of slots are wasted with padding.
B. First Fit Decreasing Height Algorithm (FFDH)

The implementation of the packing algorithm previously described is very simple but its efficiency when filling the WiMAX OFDMA downlink subframe with data could be improved. We consider now a more complex packing algorithm from the literature that could offer a higher efficiency, the so-called First Fit Decreasing Height (FFDH) algorithm [7].

The FFDH algorithm can be tailored to the WiMAX OFDMA downlink case in the following way. Given a number of bursts of known number of subchannels and symbols, bursts are placed starting from the upper left corner. A shelf is defined every time that a burst is placed on the top of the subchannel dimension. Consequent bursts are placed one after the other in the subchannel dimension direction until the next one does not fit. Then, a new shelf is created. When there is no more capacity in the current shelf to place the next burst and a new shelf can not be created, the packing algorithm terminates. Two different options could be considered for this packing algorithm, pack the bursts in the arrival order (FFDH-FIFO) or sort the bursts in non-increasing symbols order before packing (FFDH-SORTED). In the performance evaluation section we will study the pros and cons of both approaches.

In both cases though, before applying the FFDH algorithm, the dimensions of the bursts have to be determined. An example of an heuristic that could be used to find a rectangular shape for a certain amount of data to be packed is the following. Given a number of slots to be used, we find the possible rectangular shapes, considering up to a maximum padding threshold, that would result in the largest subchannel dimension and minimum padding. Figure 3 depicts an example of an FFDH packing layout.

C. Ideal Packing Algorithm

In order to be able to evaluate the performance of DL-MAP packing algorithms we define an ideal packing algorithm that gives us an upper bound on the best achievable packing efficiency. The concept behind the ideal packing algorithm is to fill, without considering any geometry limitations, all data that would fit in the capacity of the downlink subframe (fluid model). As in the previous cases, fragmentation is not considered. In this way, the ideal packing algorithm reduces the 2-dimensional packing problem to a 1-dimensional one. Note though that the geometry limitations do exist in a WiMAX OFDMA system and therefore, the efficiency results obtained with this packing algorithm should be regarded as an upper bound that can not be reached in practice.

Two additional considerations have been added in the ideal packing algorithm in order to get an upper bound closer to the actual maximum efficiency value. First, although a fluid model is considered, we account for the minimum padding that should be included for each burst as if it was a rectangular form. Second, since a WiMAX system should provide QoS guarantees, we divide the bursts to be packed in two classes: bursts that need to be transmitted in the next downlink subframe to meet the QoS agreements (Urgent) and bursts that do not
need to (Non-Urgent). The ideal scheduler packs first all urgent bursts (admission control ensures that there is no more urgent bursts than capacity), and then as many as possible of the non-urgent ones. In the following we derive the analytical expressions required to model the ideal packing algorithm.

To meet the QoS requirements we have to find a packing configuration containing at least the urgent bursts while the remaining space is filled with non-urgent bursts. Let \( U = \{u_1, \ldots, u_N\} \) and \( N = \{n_1, \ldots, n_N\} \) be the set of urgent and non urgent bursts respectively. Further, \( L \) represents the union set of \( U \) and \( N \): \( L = U \cup N \).

Now we can define \( \Gamma \) as the set of all possible packing configurations: \( \Gamma = \{\gamma \mid \gamma \in \wp(L)\} \). Here \( \wp(L) \) represents the power set of \( L \) (set of all subsets of \( L \)).

We define also \( C \) as the capacity in the WiMAX downlink subframe that can be used for data bursts and the DL-MAP: \( C = \tau \sigma - \alpha - \beta \), where \( \tau \) is the number of subchannels, \( \sigma \) the number of OFDMA symbols available in the downlink part of the WiMAX frame, and \( \alpha \) and \( \beta \) are the sizes of the frame control header and the UL-MAP, respectively.

Furthermore, we define two functions representing the burst size of a burst \( s: L \rightarrow \mathbb{N} \) and the size of DL-MAP capable of referencing the given packing configuration \( m: \Gamma \rightarrow \mathbb{N} \).

To take into consideration that bursts might produce padding even in the optimal case, we introduce a lower bound for padding, which assigns for each burst of a given size \( s \) a numbers of padded symbols \( p(s) \) as follows:

\[
p(x) = \min\{K_{i,j} \mid K_{i,j} \geq x \land 1 \geq i \geq \tau \land 1 \geq j \geq \sigma\} - x
\]

where \( K = (1 \ 2 \ \cdots \ \tau)^T \otimes (1 \ 2 \ \cdots \ \sigma) \) is the Kronecker product which represents all possible rectangular shapes fitting into the downlink subframe\(^1\).

Now, an ideal packing algorithm that tries to maximize the usage of the WiMAX downlink subframe should find a packing configuration that maximizes the sum of packed burst sizes \( S \) while guaranteeing that all urgent bursts are successfully placed:

\[
S = \max \left\{ \sum_{x_i \in \gamma} s(x_i) \mid \gamma \in \Gamma \land U \subseteq \bigcup_{x \in \gamma} x \land (2) \left( C - m(\gamma) - \sum_{x_i \in \gamma} s(x_i) + p(s(x_i)) > 0 \right) \right\}
\]

The relation ensures that all urgent bursts are packed while the inequality guarantees that the size of the packed bursts together with the DL-MAP does not exceed the space available in the downlink part of the frame. Figure 4 illustrates the way the packing problem is considered by the ideal packing algorithm.

As a result, the proposed ideal packing algorithm provides an upper bound on the maximum achievable resource usage by a DL-MAP packing algorithm that would take into account the QoS and padding conditions. Note that although this algorithm maximizes the usage of the downlink subframe resources, the solution found does not need to be the one that maximizes the throughput of the system since the combination of another set of bursts using a more efficient modulation and coding scheme could result in a larger throughput.

The ideal packing algorithm though can be easily modified to maximize the throughput instead of the packing efficiency by replacing \( \sum_{x_i \in \gamma} s(x_i) \) by \( \sum_{x_i \in \gamma} t(s(x_i)) \) in equation (2), where \( t(s(x_i)) \) corresponds to the throughput of \( s(x_i) \). The two versions of the ideal packing algorithm will be used in the next section according to the performance metric considered.

### IV. PERFORMANCE EVALUATION OF DL-MAP PACKING ALGORITHMS

In this section we evaluate through simulations the performance differences to be expected in a WiMAX PUSC 10MHz system when using one of the DL-MAP packing algorithms described in Section III (Simple, FFDH-FIFO and FFDH-SORTED) as compared to the upper bound obtained with our

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\(^1\)Consider a WiMAX downlink subframe with dimensions of \( \tau = 30 \) and \( \sigma = 16 \), PUSC 10MHz. Now, it is impossible to find a rectangular shape for a burst with a size of 31 that fits into the empty frame without padding. Therefore, in equation (1) we search for the rectangular shape that requires minimum padding. In this case it is a shape with surface size 32, i.e., 1 symbol of padding.
proposed ideal algorithm. In order to do so, we designed the following custom simulator.

We divide the input provided to a DL-MAP packing algorithm in two classes: bursts which compulsorily have to be placed in the downlink subframe in order to meet the agreed QoS requirements (Urgent) and those which do not need to (Non Urgent). To cover the different cases of ratio between Urgent and Non Urgent offered load, an admission control procedure is implemented that generates Urgent data until a percentage (specified as input parameter) of the total downlink subframe capacity is reached. Five different values are considered in our study: 20, 50, 70, 80 and 90%. Regarding the Non Urgent data, bursts are generated for the remaining of the capacity not allocated to urgent data up to 120% of the total capacity of the downlink subframe. The reason to offer to the DL-MAP packing algorithm a load that is above the capacity is to model a realistic case where more traffic than the capacity of the network might be allowed by the admission control module. By doing this the network operator can take advantage of leftover capacity to maximize the resources utilization.

The size of the frames to be packed into bursts are determined according to a packet size distribution available in [10] which has been obtained from real traces of an ISP. Moreover, the modulation and coding scheme (MCS) of each burst is randomly chosen according to the following distribution based on the assumption that the density of subscriber stations decreases inversely proportional to the distance to the base station: 20% 64QAM 3/4, 20% 64QAM 2/3, 20% 64QAM 1/2, 15% 16QAM 3/4, 15% 16QAM 1/2, 5% QPSK 3/4 and 5% QPSK 1/2. The results shown in the figures represent the average of all values after 1000 simulation runs. The corresponding error bars are also included.

The first performance metric we consider is the probability of successfully packing all urgent data. This is a key performance metric since a WiMAX system should be able to provide QoS guarantees. The results are shown in Figure 5. As expected, when the amount of data to be packed with QoS requirements increases, the probability that the packing algorithms manage to pack it in the WiMAX downlink subframe decreases, with the exception of the ideal algorithm which has no packing limitations. The FFDH algorithms outperform the Simple one because of their more complex packing that considers more possibilities to find a solution.

We study next, see Figure 6, the percentage of slots of the WiMAX downlink subframe that are used by the different packing algorithms. The Ideal packing algorithm provides us the achievable upper bound if no geometry limitations would be considered which is between 90 and 95%. The lower bound is obtained with the Simple algorithm which achieves an efficiency that varies between 53 and 70%. The FFDH algorithms achieve a similar efficiency that is better than the Simple one but still far from the Ideal upper bound thus, demonstrating that a significant room for improvement exists.

Finally, since a larger percentage usage of the
number of slots in the WiMAX downlink subframe does not necessarily translate in a larger throughput of the system, we analyze in Figure 7 the throughput achieved by the different approaches. In this case a significant worse performance of the FFDH-SORTED algorithm is observed as compared to the FFDH-FIFO algorithm and even the Simple one in the 20% case. The reason for the worse performance is that the FFDH-SORTED algorithm starts packing first the largest non-urgent data which usually corresponds to the less efficient modulation and coding schemes and thus, lower throughput.

V. SUMMARY & CONCLUSIONS

The capacity of a WiMAX system strongly depends on the combined performance of its base stations QoS schedulers and DL-MAP packing algorithms, which determine whether the agreed QoS guarantees are honoured and how efficiently the available radio resources are used. While the task of a WiMAX base station QoS scheduler is similar to the one of QoS schedulers of other networking technologies, the task of a DL-MAP packing algorithm is very specific to the WiMAX system. Therefore, in this paper we have focused on the study of the impact of the performance of a WiMAX OFDMA base station DL-MAP packing algorithm on the overall system performance.

Our results evaluated quantitatively the performance differences to be expected by using a simple DL-MAP packing algorithm as compared to more complex ones (FFDH-FIFO and FFDH-SORTED). Additionally, we have shown that special care has to be taken when designing a packing algorithm for such systems since operations like ordering the data to be packed by decreasing size might result in a larger utilization of the WiMAX downlink subframe resources but not necessarily in a larger throughput. Finally, the upper bound obtained with our ideal scheduler shows the potential performance improvement range that could be achieved by complex DL-MAP packing algorithms as compared to simple ones. Thus, the performance of a base station DL-MAP packing algorithm is a key element to be considered by WiMAX OFDMA equipment vendors in order to achieve differentiation.

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REFERENCES