Stochastic Properties of the Random Waypoint Mobility Model: Epoch Time, Direction Distribution, and Cell Change Rate

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Abstract— **The random waypoint model is a commonly used mobility model for simulations of wireless communication networks. In this paper, we present analytical derivations of some fundamental stochastic properties of this model with respect to: (a) the length and duration of a movement epoch, (b) the chosen direction angle at the beginning of a movement epoch, and (c) the cell change rate of the random waypoint mobility model when used within the context of cellular networks. Our results and methods can be used to compare the random waypoint model with other mobility models. The results on the movement epoch duration as well as on the cell change rate enable us to make a statement about the "degree of mobility" of a certain simulation scenario. The direction distribution explains in an analytical manner the effect that nodes tend to move back to the middle of the system area.**

Keywords— **Mobility modeling, simulation of mobile networks, analysis of mobile networks**

I. Introduction and Motivation

Mobility models are important building blocks in simulation–based studies of wireless networks. Researchers in this area can choose from a variety of models that have been developed in the wireless communications and mobile computing community during the last decades (see, e.g., [1][2][3][4][5]). Also well–known motion models from physics and chemistry, such as random walks or Brownian motion, and models from other engineering disciplines, such as transportation theory [6][7], are used in simulations of mobile networks. Surveys and classifications on this topic can be found in [3], [5], [8], and [9].

A very popular and commonly used mobility model is the *random waypoint (RWP) model*. It is implemented in the network simulation tool *ns2* [10] and is used in several performance evaluations of ad hoc networking protocols (see, e.g., [11][12][13]). This mobility model is a simple and straightforward stochastic model that describes the movement behavior of a mobile network node in a given system area as follows: A node randomly chooses a destination point p_d in the area and moves with constant speed to this point. After waiting a certain pause time t_p, it chooses a new destination and, optionally, a new speed, moves with constant speed to this destination, and so on. The movement of a node from a starting position p_s to its next destination p_d is denoted as one *movement epoch*. The destination points ("waypoints") are uniformly randomly chosen in the system area.

In previous work, one of the authors noted that the understanding of the behavior of this model is very important in order to interpret simulation results correctly [5][14][15]. Among other things, in [14] and [15] the stationary spatial node distribution was studied and information about the probability that a node is located in a certain subarea were derived. The paper on hand represents a continuation of research in this field. It gives an *analytical* discussion of some stochastic characteristics of the RWP model.

In Section II, we derive typical stochastic parameters of a node's traveled *distance* and *time* during one movement epoch by making use of some classical results of geometrical probability theory. In particular, we give equations for the expected value, the variance, and the probability density function of the epoch length on a circular and rectangular system area. Next, we discuss the mapping from epoch length to epoch duration, where we consider scenarios with and without pause time in the destination points.

We are interested in these values because they define kind of a "mobility metric" that can be used to describe a certain simulation scenario. In Section III we calculate the probability density function of the *movement direction* of a node at a given location. This direction distribution describes analytically the effect reported in [14] and [16] that RWP nodes tend to move back from the border to the middle of the area. Finally, in Section IV, we employ the RWP model for movement of mobile stations in a cellular network. We investigate the expected number of cell changes per epoch and the expected number of cell changes per unit time, i.e., the *cell change rate*.

II. Movement Epoch Length and Time

In order to compare simulation results that are done with different random mobility models, it is desired to define kind of a metric for the "degree of mobility" of the simulated scenario. Due to the broad range of mobility models used in the literature and their various parameters, such a definition is not trivial. However, we can state that two mobility parameters are of major interest in all models: the speed behavior (Is there a constant speed or a speed distribution? When and how does a node change its speed?) and the direction change behavior (What is the frequency of direction changes? How does a node change its direction?). For example, in the commonly used *random direction model* as described in [5], the time between two direction change events is taken from an exponential distribution. This time is independent of the speed of the node; and, if a wrap–around border behavior [14] is used, it is also independent of the size and shape of the area.

As opposed to this, the RWP model correlates the speed and the direction change behavior. The time between two direction change events is not anymore an adjustable input parameter of model, but depends on the speed of the nodes and the size and shape of the area. For a given area, a higher speed results in a higher frequency of direction changes.

In this section, we investigate the time between two direction change events (i.e., the movement epoch time) of the RWP model. We first define the RWP model as a stochastic process and show ergodic properties of the process. Afterward, we give an analytical expression for the probability density function (pdf) and the first two moments of the movement epoch length, i.e., the distance that a node travels during one epoch. Hereby, we first investigate an RWP model in one dimension and then consider rectangular and circular areas. Finally, the conversion from epoch length to epoch time is discussed, with and without pause time.

A. Stochastic Processes and Ergodic Properties

We describe the RWP movement of a node as a discrete–time stochastic process $\{P_i\}_{i\in\mathbb{N}_0}$ given by selecting the random waypoints P_i, $i \geq 0$. The corresponding stochastic process of distances between two consecutive waypoints is given by $\{L_i\}_{i\in\mathbb{N}}$, with $L_i = \|P_i - P_{i-1}\|$, $i \geq 1$. When we just refer to a single random variable of a process, we omit the index i and just write P or L. A sample of process $\{L_i\}$ is denoted by $\{l_i\}$.

While the random waypoints are independent and identically distributed (i.i.d.) per definition, the random distances are not stochastically independent, essentially because the endpoint of one movement epoch is the starting point of the successive movement epoch.

In order to derive the average movement epoch length

$$\lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} l_i$$

observed by a single RWP node, we show for $N \to \infty$ that

$$\frac{1}{N} \sum_{i=1}^{N} l_i \overset{N\to\infty}{\longrightarrow} E(L) \quad \text{almost everywhere,} \tag{1}$$

i.e., for all sample sequences up to a 'null set.' In other words, the 'ensemble average' $E(L)$ of one epoch of an RWP process with many nodes is equal to the long–run 'time average' $\frac{1}{N}\sum_{i=1}^{N} l_i$ of a RWP process of a single node. In the nomenclature of random processes, we thus show the 'mean–ergodic property' of the RWP mobility model.

While this result is intuitively convincing, the proof is not totally trivial since the random variables $L_1, L_2, L_3, \ldots$ are not stochastically independent. But despite the fact that 'last endpoint equals next starting point,' the ergodic property holds as can be seen as follows. Assume we look at the random process given by $\{L_{2i-1}\}_{i\in\mathbb{N}}$, i.e., we look only at every second variable of the original process of random distances. Now this subset of variables is independent and identically distributed, thus, the mean ergodic property here is obtained immediately. The same is true for the sub–process $\{L_{2i}\}_{i\in\mathbb{N}}$. Now, combining these two sub–processes does not change the asymptotic behavior of the time averages of the 'combined' process since the union of two null sets is still a null set, thus, Equation (1) holds.

In the same manner one also obtains the 'distribution–ergodic property' of the process, i.e., statistically there is no difference of sampling repeatedly from a single random variable L or successively from sequence $\{L_i\}_{i\in\mathbb{N},i\geq 1}$.

With respect to our problem to determine the average movement epoch length and time observed by a single node, the above mentioned ergodic properties imply the following: in order to determine the distribution or expected value of the epoch lengths a single node following the RWP model observes, the analysis can be simplified by considering only the distribution of the distance between two points randomly placed in the system area. This observation allows the transfer of some results of geometrical probability theory to our problem.

In the following, the random variable S describes the distance between two independent random points in the system area sampled from a uniform distribution. The distance between two points p_1 and p_2 is denoted as $s = \| p_2 - p_1 \|$.

B. Epoch Length on One–Dimensional Line

We first consider a one–dimensional line segment $[0, a]$. Two random points are independently uniformly placed on this segment, i.e., the pdf of a point's location $P = X$ is

$$f_X(x) = \frac{1}{a} \quad \text{for } 0 \le x \le a,$$

and 0 otherwise. The distance between two random points is defined by $S = |X_2 - X_1|$.

The pdf of this random distance under the condition that one point lies at $X = x_1$ can be written as

$$f(s \mid x_1) \propto \begin{cases} 2 & \text{for } 0 \le s \le x_1 \\ 1 & \text{for } x_1 < s \le a - x_1 \\ 0 & \text{else} \end{cases}$$

for $x_1 \le a/2$ and

$$f(s \mid x_1) \propto \begin{cases} 2 & \text{for } 0 \le s \le a - x_1 \\ 1 & \text{for } a - x_1 < s \le x_1 \\ 0 & \text{else} \end{cases}$$

for $x_1 > a/2$. Using the Heaviside unit step function

$$u(s - s_0) = \begin{cases} 1 & \text{for } s > s_0 \\ 0 & \text{for } s < s_0 \end{cases},$$

we obtain in both cases

$$f(s \mid x_1) \propto 2\,u(s) - u(s - x_1) - u(s - a + x_1).$$

Integration over all possible x_1 values and proper normalization yields the desired pdf of the distance as

$$f_S(s) = -\frac{2}{a^2}\, s + \frac{2}{a} \tag{2}$$

for $0 \le s \le a$ and 0 otherwise. The expected distance is

$$E(S) = \int_0^a s\, f(s)\, ds = \frac{1}{3}\, a, \tag{3}$$

and its variance can be calculated as

$$E(S^2) = \int_0^a s^2\, f(s)\, ds = \frac{1}{6}\, a^2. \tag{4}$$

With our thoughts above, the substitution $(S, s) \to (L, l)$ in (2), (3), and (4) yields the pdf $f_L(l)$, mean $E(L)$, and variance $E(L^2)$ of the movement epoch length L of a one–dimensional RWP model.

C. Epoch Length on Rectangular System Area

Let us now consider an RWP movement on a rectangular area of size $a \times b$ and derive the distribution of the movement epoch length L. To do so, we must derive the distribution $f_S(s)$ of the distance between two independent random points. Again, we use a uniform placement of the destination points, i.e.,

$$f_{XY}(x, y) = \frac{1}{ab} \quad \text{for } 0 \le x \le a \text{ and } 0 \le y \le b.$$

The Euclidian distance between two points $p_1 = (x_1, y_1)$ and $p_2 = (x_2, y_2)$ within a rectangular area is

$$\begin{aligned} s = \| p_2 - p_1 \| &= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \\ &= \sqrt{s_x^2 + s_y^2}. \end{aligned}$$

The random variables $S_x = |X_2 - X_1|$ and $S_y = |Y_2 - Y_1|$ are independent and their pdf is given by (2). Thus, the joint pdf of S_x and S_y is

$$f_{S_X S_Y}(s_x, s_y) = \frac{4}{a^2 b^2}(-s_x + a)(-s_y + b)$$

for $0 \le s_x \le a$ and $0 \le s_y \le b$. From this, we can derive the pdf $f_S(s)$ of $S = \sqrt{S_x^2 + S_y^2}$ as published in [17]; and, with this result, we can state the following theorem for the RWP model.

Theorem 1 (Length of movement epoch on a rectangular area): The probability density function of the length L of a movement epoch in the random waypoint model on a rectangular area of size $a \times b$, $a \ge b$, is

$$f_L(l) = \frac{4l}{a^2 b^2} \cdot f_0(l) \tag{5}$$

with

$$f_0(l) = \begin{cases} \frac{\pi}{2} ab - al - bl + \frac{1}{2} l^2 \\ \quad \text{for } 0 \le l \le b \\ ab \arcsin \frac{b}{l} + a\sqrt{l^2 - b^2} - \frac{1}{2} b^2 - al \\ \quad \text{for } b < l < a \\ ab \arcsin \frac{b}{l} + a\sqrt{l^2 - b^2} - \frac{1}{2} b^2 - \\ ab \arccos \frac{a}{l} + b\sqrt{l^2 - a^2} - \frac{1}{2} a^2 - \frac{1}{2} l^2 \\ \quad \text{for } a \le l \le \sqrt{a^2 + b^2} \end{cases}, \tag{6}$$

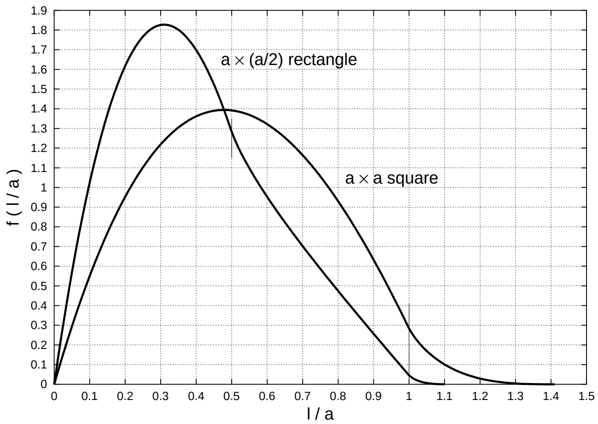

Fig. 1. Pdf of epoch length L on a rectangle

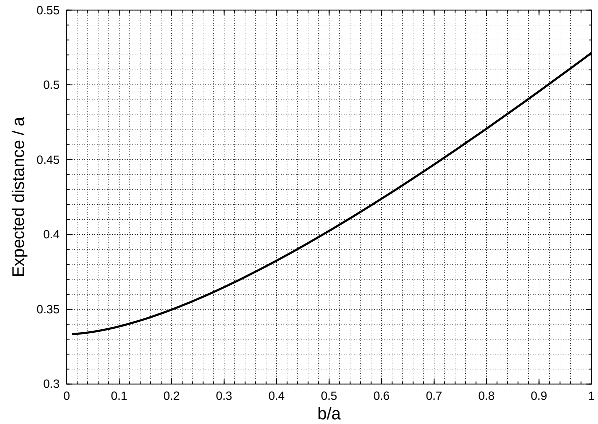

Fig. 2. Expected epoch length within an $a \times b$ rectangle

and 0 otherwise. □

Figure 1 shows $f_{L^*}(l^*)$ with $l^* = l/a$ for areas of size $a \times a$ and $a \times \frac{a}{2}$. For arbitrary a, the value of $f_L(l)$ is obtained by $f_L(l) = \frac{1}{a} f_{L^*}(l^*)$. The expected value of L is

$$E(L) = \frac{1}{15}\left[\frac{a^3}{b^2} + \frac{b^3}{a^2} + \sqrt{a^2+b^2}\left(3 - \frac{a^2}{b^2} - \frac{b^2}{a^2}\right)\right] + \frac{1}{6}\left[\frac{b^2}{a}\operatorname{arcosh}\frac{\sqrt{a^2+b^2}}{b} + \frac{a^2}{b}\operatorname{arcosh}\frac{\sqrt{a^2+b^2}}{a}\right] \quad (7)$$

with $\operatorname{arcosh}(x) = \ln\left(x + \sqrt{x^2-1}\right)$. Figure 2 shows the curve for $E(L)/a$ over b/a. For example, the expected distance of two random points within a square of size $a \times a$ is $E(L) = 0.521a$, and a rectangle of size of $a \times \frac{a}{2}$ yields $E(L) = 0.402a$. The variance of L is given by

$$E(L^2) = \frac{1}{6}\left(a^2 + b^2\right) \ . \quad (8)$$

Note that for $b \to 0$ the moments for the one dimensional case are obtained, i.e., $\lim_{b\to 0} E(L) = \frac{1}{3}\, a$ and $\lim_{b\to 0} E(L^2) = \frac{1}{6}\, a^2$.

We simulated an RWP node over $N = 10^7$ epochs and measured the traveled distance of each epoch. The resulting sample mean $\hat{E}(L) = \frac{1}{N}\sum_{i=1}^{N} l_i$ is shown for various a and b in Table I along with the analytical value $E(L)$. We clearly observe that the simulated values accurately match the analytical results.

TABLE I

EXPECTED EPOCH LENGTH ON RECTANGULAR AREA

a	b	$\hat{E}(L)$	$E(L)$
100	1	33.328	33.342
100	20	34.977	34.976
100	50	40.237	40.239
100	100	52.137	52.141
200	100	80.486	80.477
200	200	104.273	104.281
500	500	260.710	260.702
1000	500	402.373	402.386
1000	1000	521.403	521.405

D. Epoch Length on Circular System Area

Finally, we consider a circular system area of radius R. The Euclidian distance between two points $p_1 = (r_1,\, \phi_1)$ and $p_2 = (r_2,\, \phi_2)$ within this disk is

$$s = \|\, p_2 - p_1 \| = \sqrt{r_1^2 + r_2^2 - 2 r_1 r_2 \cos\left(\phi_2 - \phi_1\right)} \ .$$

The random destination points have a uniform pdf

$$f(r,\, \phi) = \frac{1}{R^2 \pi} \quad \text{for } 0 \le r \le R \text{ and } 0 \le \phi < 2\pi \ ,$$

which can be achieved by a uniformly distributed angle ϕ and

$$f(r) = \frac{2r}{R^2} \quad \text{for } 0 \le r \le R \ .$$

The pdf of the distance S between two independent uniform random points within a disk of radius R is given in [18][19], and we can state the following theorem.

Theorem 2 (Length of movement epoch on disk): The probability density function of the length L of a movement epoch in the random waypoint model on a disk of radius R is given by

$$f_L(l) = \frac{8}{\pi R}\frac{l}{2R}\left(\arccos\frac{l}{2R} - \frac{l}{2R}\sqrt{1 - \left(\frac{l}{2R}\right)^2}\right) \ , \quad (9)$$

for $0 \le l \le 2R$ and 0 otherwise. □

Figure 3 shows the plot of $f_{L^*}(l^*)$ with $l^* = l/2R$. The

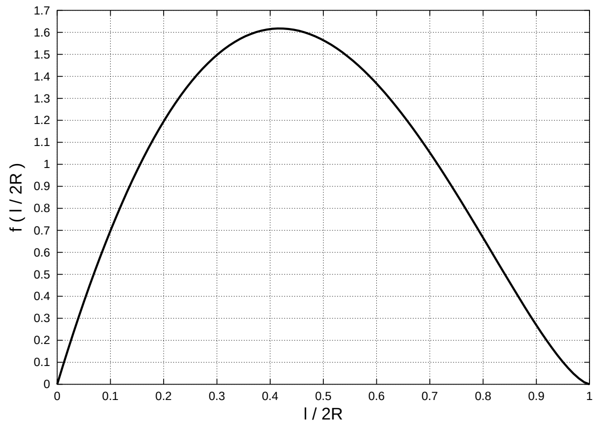

Fig. 3. Pdf of epoch length L on a disk with radius R

expected value of L is

$$E(L) = \int_0^{2R} l\, f(l)\, dl = \frac{128}{45\,\pi} \cdot R\,, \tag{10}$$

and its variance is

$$E(L^2) = \int_0^{2R} l^2\, f(l)\, dl = R^2. \tag{11}$$

E. Movement Epoch Time

Let us now employ our results on the epoch length to calculate the stochastic properties of the epoch time.

The random variable T denotes the time that a node moves during one epoch without any pause time in the destination point. If the speed of a node, denoted as v, is constant during the entire movement process and $v > 0$, we have

$$T = \frac{1}{v}\, L.$$

Hence, the pdf of the movement epoch time is given by

$$f_T(t) = v\, f_L(v\, t)\,, \tag{12}$$

and the expected time is

$$E(T) = \frac{1}{v}\, E(L)\,, \tag{13}$$

with f_L and $E(L)$ taken from (6) and (7), or (9) and (10), respectively.

If the speed v of a node is not constant but chosen from a distribution $f_V(v)$ at the beginning of each epoch (and then stays constant during this epoch), we have

$$T = \frac{L}{V}$$

with

$$f_T(t) = \int_0^{\infty} v\, f_L(vt)\, f_V(v)\, dv \quad \text{for } t \geq 0\,. \tag{14}$$

Let us now consider the general case in which a node rests a certain pause time t_p, which is taken from a general distribution $f_{T_p}(t_p)$ with $f_{T_p}(t_p) = 0$ for $t_p < 0$ and mean $E(T_p)$. The random variable for the total epoch time, including the pause time at the destination point is given by

$$T' = T + T_p.$$

This linear combination of the two independent random variables T and T_p yields the pdf

$$f_{T'}(t') = \int_0^{t'} f_T(t)\, f_{T_p}(t' - t)\, dt \quad \text{for } t' \geq 0 \tag{15}$$

and the expected movement epoch time

$$E(T') = E(T) + E(T_p). \tag{16}$$

F. Example

Given is a simulation scenario on a $1000 \times 500\ \text{m}^2$ system area. The nodes move according to the RWP model with constant speed $v = 10\,\text{m/s}$ and a pause time $t_p > 0$ taken from a negative exponential distribution $f_{T_p}(t_p) = \mu e^{-\mu t_p}$ with a mean pause time $\mu = 10$ s. The expected epoch length is $E(L) = 402\,\text{m}$. The expected epoch time is $E(T') = 40.2\ \text{s} + 10\ \text{s} = 50.2$ s. If we increase the speed to $v = 20$ m/s, we obtain $E(T') = 20.1\ \text{s} + 10\ \text{s} = 30.1$ s, i.e., a node is expected to change its direction more frequently. Increasing the system area to $1000 \times 1000\ \text{m}^2$ reduces the frequency of direction changes to $E(T') = 62.1$ s (with $v = 10$ m/s) or $E(T') = 36.05$ s (with $v = 20$ m/s).

This example illustrates that, in the RWP model, the mobility metric 'speed' and the size and shape of the area directly influence the mobility metric 'direction change.' The two metrics 'speed' and 'direction change' cannot be treated as independent input parameters in this model.

III. Movement Direction

Let us now regard a node who is just to begin a new movement epoch. It is located at p_s and must choose a new destination point p_d from a uniform distribution. As one of the authors noted in [14], one characteristic of the random waypoint model is that nodes whose movement epoch starts close to the border of the simulation area tend to move back toward the middle of the area in this epoch. This observation is also reported in [16] ("density waves"). For example, a node located at $p_s = (r_s, \phi_s) = (\frac{R}{2},\ 0)$ on a circular system area with radius R finds more destination points in the directions toward the center of the area (to the left) than in the directions toward the border (to the right). Among all possible directions $[0 \dots 2\pi[$, the one that requires the node to pass the center of the area during the next movement epoch has the highest probability to be "chosen." In summary, we can say that the movement direction of a random waypoint node is not uniformly distributed and its pdf depends on the coordinates of the current starting point p_s. In the following, we show how to derive this pdf.

A. One–Dimensional Line

First, we consider a one–dimensional line segment $[-a, a]$. A node that is located at a given starting point x_s chooses a new destination point, and then moves either to the right (direction $\gamma_s = 0$) or to the left (direction $\gamma_s = \pi$).

It is straightforward to observe that a certain direction is chosen with high probability, if many destination points lie in this direction. Since a destination point is taken from a uniform distribution, we obtain $P(\gamma_s = 0 \mid x_s) = \frac{2a - x_s}{2a}$ and $P(\gamma_s = \pi \mid x_s) = \frac{x_s}{2a}$, and therefore

$$f(\gamma_s \mid x_s) = \frac{2a - x_s}{2a}\,\delta(\gamma_s) + \frac{x_s}{2a}\,\delta(\gamma_s - \pi)\,. \tag{17}$$

The function $\delta(x)$ denotes the Dirac delta function, which has the properties $\delta(x) = 0$ for $x \neq 0$ and $\delta(0) = \infty$ and $\int_{-\infty}^{\infty} \delta(x)dx = 1$.

B. Circular System Area

In the following, we derive this pdf as a function of $p_s = (r_s,\ \phi_s)$ for a circular system area with radius R.

The definition of a node's direction γ_s is shown in Figure 4. This direction angle remains constant during one movement epoch, i.e., $\gamma = \gamma_s$. In addition, we introduce a second direction angle at point p_s, denoted as θ_s, which is independent of ϕ_s and can be directly mapped to γ_s for given ϕ_s. Its definition is also shown in Figure 4. If we define that clockwise angles count negative and counterclockwise angles count positive, we have

$$\gamma_s = \pi + \phi_s + \theta_s.$$

Without any calculation, we can already make the following statements about the direction distribution. First, the pdf is independent of ϕ_s, i.e.,

$$f(\theta_s \mid (r_s, \phi_s)) = f(\theta_s \mid r_s).$$

Second, the direction is uniformly distributed for $r_s = 0$, i.e.,

$$f(\theta_s \mid r_s = 0) = \frac{1}{2\pi} \quad \text{for} \quad 0 \le \theta_s < 2\pi.$$

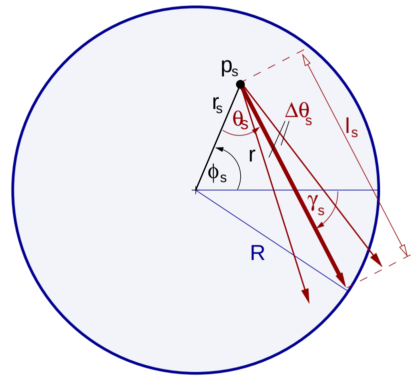

Fig. 4. Definitions of movement angles and lengths

Third, for all other $r \neq 0$, the highest value in the pdf is achieved for $\theta_s = 0$, the lowest for $\theta_s = \pi$, i.e.,

$$\begin{aligned} \max(f(\theta_s \mid r_s)) &= f(\theta_s = 0 \mid r_s) \qquad \text{for} \quad 0 < r_s < R \\ \min(f(\theta_s \mid r_s)) &= f(\theta_s = \pi \mid r_s) \qquad \text{for} \quad 0 < r_s < R. \end{aligned}$$

Last but not least, the pdf shows the symmetry

$$f(\theta_s - \Delta\theta_s \mid r_s) = f(\theta_s + \Delta\theta_s \mid r_s) \quad \forall \Delta\theta_s, \forall r_s\,.$$

Let $P(\theta_s - \Delta\theta_s \le \Theta_s \le \theta_s + \Delta\theta_s)$ denote the probability that a node at position (r_s, ϕ_s) chooses a movement angle within the interval $\Delta\theta_s$ around a certain θ_s (see Fig. 4). A node chooses a certain direction within the interval $\theta_s \pm \Delta\theta_s$ if its destination point lies with the area spanned by $\theta_s \pm \Delta\theta_s$. Let us denote this area by A_Δ. Since a destination point is chosen from a uniform distribution, we can set up

$$P(\theta_s - \Delta\theta_s \le \Theta_s \le \theta_s + \Delta\theta_s) = \frac{A_\Delta}{R^2\pi}.$$

Our aim is to determine the pdf

$$f(\theta_s \mid r_s) = \lim_{\Delta\theta_s \to 0} \frac{P(\theta_s - \Delta\theta_s \le \Theta_s \le \theta_s + \Delta\theta_s)}{2\,\Delta\theta_s}.$$

For small $\Delta\theta_s$, we can approximate the spanned area by two triangular areas, which yields

$$A_\Delta \;\approx\; 2 \cdot \frac{1}{2} \cdot l_s \cdot l_s \tan\Delta\theta_s \;\approx\; l_s^2 \Delta\theta_s, \tag{18}$$

where $l_s = l_s(\theta_s,\ r_s)$ denotes the Euclidian distance from the position $p_s = (r_s,\ \phi_s)$ to the border of the area $r = R$ in this direction; see Fig. 4. Note that l_s is independent of ϕ_s. Combining the above three equations yields

$$f(\theta_s \mid r_s) = \frac{1}{2R^2\pi}\, l_s^2. \tag{19}$$

Using the law of cosines in Fig. 4, we obtain a quadratic equation for l_s, whose solution is

$$l_s = r_s \cos\theta_s + \sqrt{r_s^2 \cos^2\theta_s - r_s^2 + R^2}, \tag{20}$$

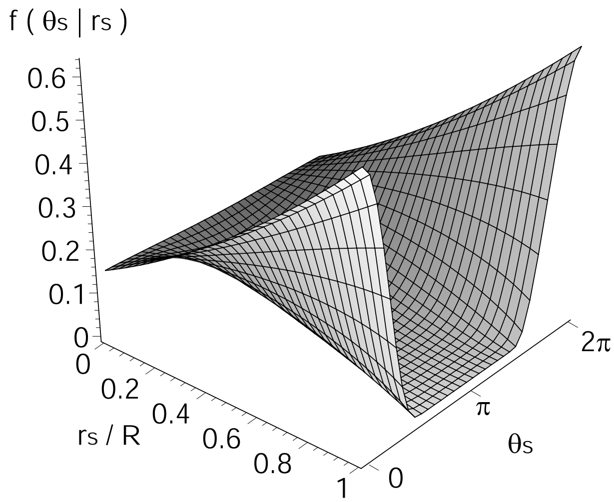

Fig. 5. Distribution of the movement direction $f(\theta_s \mid r_s)$

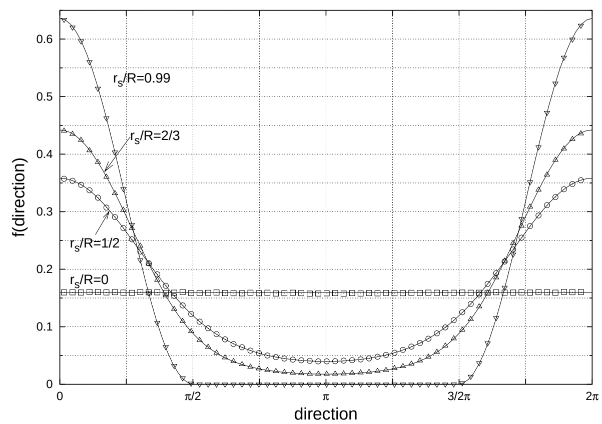

Fig. 6. Distribution of the movement direction $f(\theta_s \mid r_s)$. Dots are simulation results, lines are analytical curves.

which allows us to conclude with the following theorem.

Theorem 3 (Movement direction on disk): A node moves on a circular area of radius R according to the random waypoint model. At the beginning of each movement epoch, it chooses a new destination point. The probability density function of the direction θ_s (defined in Fig. 4) toward this destination point is

$$f(\theta_s \mid r_s) = \frac{1}{2\pi}\left(\frac{r_s}{R}\cos\theta_s + \sqrt{\frac{r_s^2}{R^2}\left(\cos^2\theta_s - 1\right) + 1}\right)^2 \tag{21}$$

where (r_s, ϕ_s) denotes the position at the beginning of the epoch. The plot of $f(\theta_s \mid r_s)$ is shown in Fig. 5 for various r_s. The direction $\phi = \phi_s$ of this movement epoch is given by $\gamma_s = \pi + \phi_s + \theta_s$ (see Fig. 5). □

We run a number of simulations in which a node is positioned at a given r_s/R and the chosen directions are recorded. The percentage of directions within the interval $[\theta_s, \theta_s + 0.1]$ serves as an estimate for $f(\theta_s + 0.05 \mid r_s)$. Fig. 6 shows that the simulation results match the analytical curves well.

Let us briefly discuss the direction distribution. We observe that the above statements on the symmetry and maxima/minima hold. Furthermore, we observe that $f(\theta_s \mid r_s)$ just depends on the ratio r_s/R. Nodes located at the border of the area, i.e., r_s close to R, have a high probability to choose θ_s around 0, i.e., they tend to move toward the center of the disk. In the extreme case $r \to R$, we have $f(\theta_s \mid r) = 0$ for $\frac{\pi}{2} < \theta_s < \frac{3}{2}\pi$. This effect decreases as a node starts closer to the center of the disk, until a uniform direction distribution is achieved for $r_s = 0$.

C. Rectangular System Area

The calculation of the pdf on a rectangular system area, i.e., $f(\theta_s \mid (x_s, y_s))$, could be performed in a similar manner. However, the derivation is more complicated than in the circular case, because l_s depends on both coordinates of the starting point (x_s and y_s). The derivation for a rectangular system area represents an issue for future work.

IV. Cell Change Rate

The RWP model is now applied to model the movement of mobile stations in a cellular–structured network, e.g., in a simulation of a wireless LAN system. Our aim is to make a statement about the expected cell change rate (i.e., the number of cell changes per second) of an RWP node within a rectangular system area of size $a \times b$. This rate, in combination with a teletraffic model, is required to estimate the handover rate of a node. On the way to our solution, we first derive the expected number of cell changes per movement epoch, denoted as $E(C)$, and then combine this result with the results of Section II on the expected epoch time $E(T) = \frac{1}{v}\,E(L)$.

We first consider an area consisting of two cells of equal size. In a second step, we consider the case of having $\alpha \times \beta$ cells of equal size $\frac{a}{\alpha} \times \frac{b}{\beta}$.

Many results concerning cell change rates are published with other mobility models, but, to the best of our knowledge, the cell change rate of the RWP model has not been investigated yet.

A. Cell Change Rate for Two Cells

Let us follow a single node in a scenario as depicted in Fig. 7. We want to analyze how frequently on average the node crosses the cell boundary depending on the speed v of the node.

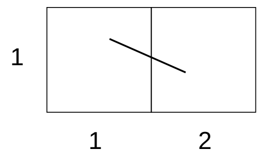

Fig. 7. Example for cell changes with two cells

The probability for the events 'next random waypoint is in same cell as previous one' and 'next random waypoint is in the other cell compared to previous one' is 1/2 each, since the cells are of same size. Thus, on average there is one cell change per two movement epochs, i.e., $E(C) = \frac{1}{2}$. We therefore obtain as result for a node moving at constant speed v that the average cell change rate of a single node is

$$\frac{E(C)}{E(L)/v} = \frac{1}{2}\frac{v}{E(L)}, \tag{22}$$

where the average length of a movement epoch, $E(L)$, was presented in (7).

B. Generalization to More Cells

In the following we assume that the system area of size $a \times b$ consists of $\alpha \times \beta$ cells of equal size $\frac{a}{\alpha} \times \frac{b}{\beta}$. The x–position of a cell is indexed as $x = 1, \ldots, \alpha$, and its y–position as $y = 1, \ldots, \beta$.

In order to generalize the result obtained in the previous section to the case of more cells, we first have to compute the expected number $E(C)$ of cell boundary crossings per movement epoch.

B.1 Cell Changes per Epoch

We denote the number of cell changes that a node performs during a movement epoch between two given cells (x_1, y_1) and (x_2, y_2) by $c(^{x_2 y_2}_{x_1 y_1})$. In order to calculate $E(C)$, we must know for each cell pair the value $c(^{x_i y_i}_{x_j y_j})$, i.e., how many cell changes are caused by a movement between these cells. The value of $E(C)$ can then be computed via

$$E(C) = \frac{1}{\alpha^2 \beta^2} \sum_{x_j=1}^{\alpha} \sum_{y_j=1}^{\beta} \sum_{x_i=1}^{\alpha} \sum_{y_i=1}^{\beta} c(^{x_i y_i}_{x_j y_j}). \tag{23}$$

Because of our 'idealized' rectangular cell shapes, the question arises how to count the number of cell changes for diagonal moves. We will use the metrics derived from the l_1 and l_∞ norms to provide upper and lower bounds on the number of cell changes as follows.

Upper bound: If we assume that a node moving to a diagonally adjacent cell experiences *two* cell changes during this movement (see Fig. 8a), the number of cell changes for a movement epoch between two cells (x_i, y_i) and (x_j, y_j) can be calculated as the Manhattan distance (l_1 norm) between the cells, i.e.,

$$c_1(^{x_j y_j}_{x_i y_i}) = |x_i - x_j| + |y_i - y_j| \ . \tag{24}$$

Lower bound: If we assume that a diagonal move causes only *one* cell change (see Fig. 8b), the number of cell changes for a movement epoch between two cells (x_i, y_i) and (x_j, y_j) is calculated as the chessboard distance (l_∞ norm) between the cells, i.e.,

$$c_\infty(^{x_j y_j}_{x_i y_i}) = \max\{|x_i - x_j|, |y_i - y_j|\} \ . \tag{25}$$

To summarize, we can state that if we apply the RWP model in a system area with $(\alpha \times \beta)$ cells of equal size, we can estimate the expected number $E(C)$ of cell changes in one movement epoch by

$$E(C_\infty) \le E(C) \le E(C_1) \quad \text{with} \tag{26}$$

$$E(C_\infty) = \frac{1}{\alpha^2 \beta^2} \sum_{x_i,\, x_j=1}^{\alpha} \sum_{y_i, y_j=1}^{\beta} \max\{|x_i - x_j|, |y_i - y_j|\}$$

$$E(C_1) = \frac{1}{\alpha^2 \beta^2} \sum_{x_i, x_j=1}^{\alpha} \sum_{y_i, y_j=1}^{\beta} (|x_i - x_j| + |y_i - y_j|) \, .$$

For high values of α and β, say $\alpha, \beta > 10$, we can make the following approximations:

$$E(C_\infty) \approx \frac{7}{15}\,\alpha\,; \beta = \alpha\,; \quad E(C_\infty) \approx \frac{20}{27}\,\alpha\,; \beta = 2\alpha$$

$$\text{and } \ E(C_1) \approx \frac{1}{3}\,(\alpha + \beta)\,.$$

Example: We give a simple example for a scenario with 3×3 cells (see Fig. 8). To compute the upper bound, a movement epoch from cell $(1,1)$ to $(1,1)$ yields no cell change, an epoch from $(1,2)$ to $(1,1)$ or vice versa yields one cell change, between $(2,2)$ to $(1,1)$ we obtain two

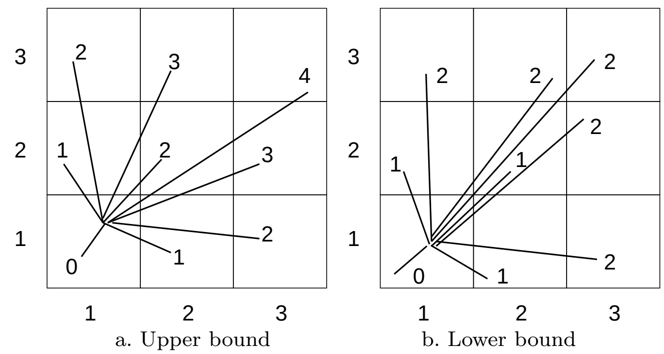

Fig. 8. Example for cell changes

changes, and so on. The upper bound $E(C_1)$ of expected cell changes per movement epoch, computed using (23) with $c(\substack{x_j y_j \\ x_i y_i})$ replaced by $c_1(\substack{x_j y_j \\ x_i y_i})$, equals 1.78. If we assume that a movement between diagonal adjacent cells causes only one cell change, the corresponding $E(C_\infty)$ equals 1.28. Thus, we have $1.28 \leq E(C) \leq 1.78$.

B.2 Cell Change Rate

Finally, along the lines of the derivation of expression (22) we can obtain the cell change rate for a node via

$$\frac{E(C) \cdot v}{E(L)}, \tag{27}$$

where v denotes again the speed of the node and $E(L)$ denotes the expected length of a movement epoch given in Section II. Expression (27) can be estimated using the upper and lower bounds for $E(C)$ as indicated above.

V. Conclusions

This paper presented an analytical study of some stochastic properties of the well–known random waypoint mobility model. We investigated (a) the length and duration of a movement epoch, (b) the chosen direction angle at the beginning of a movement epoch, and (c) the number of cell changes. These results make it possible to compare the RWP model with other mobility models.

The results on the movement epoch length and duration well as the cell change rate enable us to make a statement about the "degree of mobility" of a certain simulation scenario. This is needed if we want to compare simulation results made with the RWP model and a different model. We have shown that the time between two direction changes is determined by the speed of the nodes v and size and shape of the area.

Furthermore, this investigation gives a deeper understanding on how the RWP model behaves and outlines pitfalls that might occur if we are not aware of this behavior. For example, the derived direction distribution explains in an analytical manner the effect reported in [14] and [16] that nodes tend to move back to the middle of the area.

Last but not least, our methods can also be applied to other mobility models, in order to derive appropriate measures which describe mobility models in an precise manner.

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